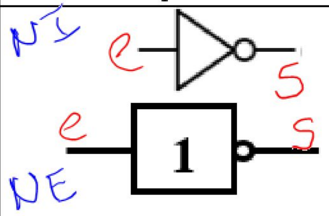
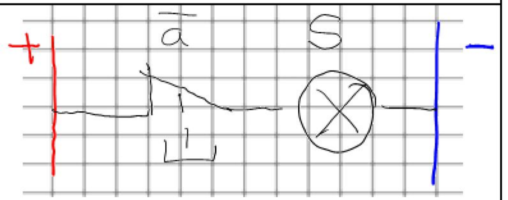
Les fonctions logiques :

<table>
<tr><th>Fonction</th><th>Symbole</th><th>Table de vérité</th><th>Equation</th><th>Schéma à contacts</th></tr>
<tr><td>NON</td><td></td><td><table><tr><th>a</th><th>S</th></tr><tr><td>0</td><td>1</td></tr><tr><td>1</td><td>0</td></tr></table></td><td>S = $\bar{a}$</td><td></td></tr>
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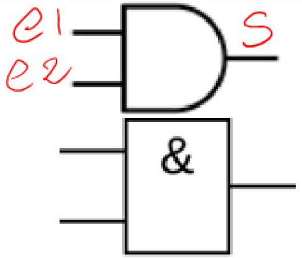
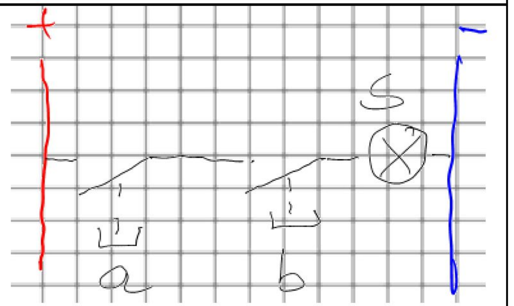
and</td><td></td><td><table><tr><th>a</th><th>b</th><th>S</th></tr><tr><td>0</td><td>0</td><td>0</td></tr><tr><td>0</td><td>1</td><td>0</td></tr><tr><td>1</td><td>1</td><td>1</td></tr><tr><td>1</td><td>0</td><td>0</td></tr></table></td><td>S = a.b</td><td></td></tr>
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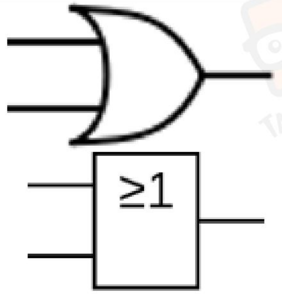
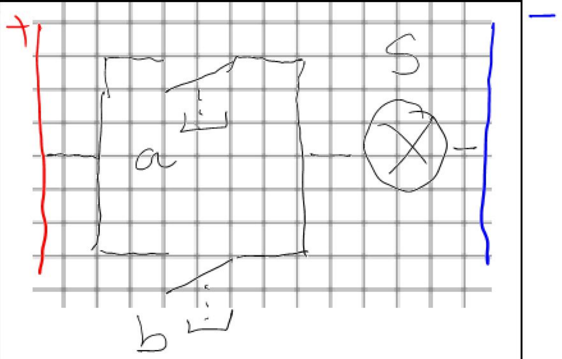
OR</td><td></td><td><table><tr><th>a</th><th>b</th><th>S</th></tr><tr><td>0</td><td>0</td><td>0</td></tr><tr><td>0</td><td>1</td><td>1</td></tr><tr><td>1</td><td>1</td><td>1</td></tr><tr><td>1</td><td>0</td><td>1</td></tr></table></td><td>S = a+b</td><td></td></tr>
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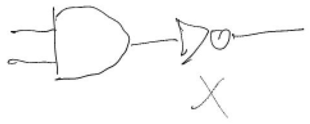
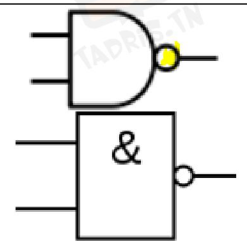
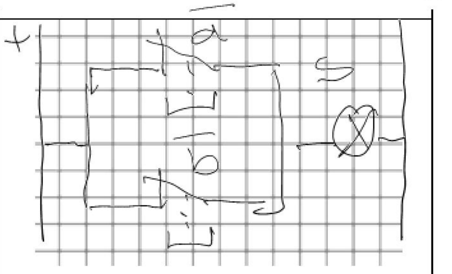
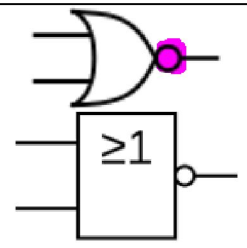
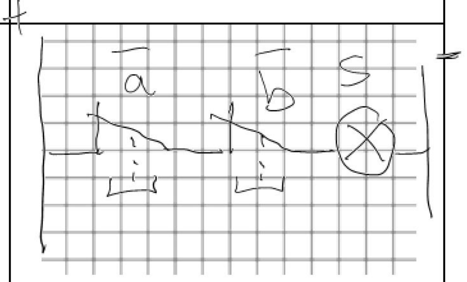
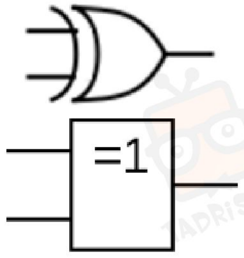
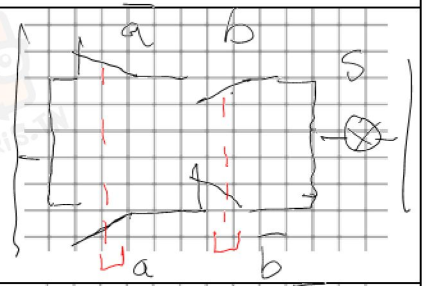
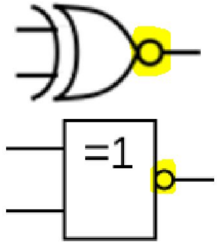
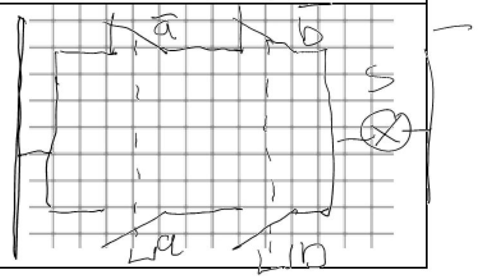


NAND non et		a b S 0 0 1 0 1 1 1 1 0 1 0 1	$= \overline{a}+\overline{b}$ S = $\overline{a \cdot b}$ $= a \mid b$	
non ou **NI** NOR		a b S 0 0 1 0 1 0 1 1 0 1 0 0	$= \overline{a} \cdot \overline{b}$ S = $\overline{a+b}$ $= a \downarrow b$	
OU exclusif XOR		a b S 0 0 0 0 1 1 1 1 0 1 0 1	S = $a \oplus b$ $= \overline{a} \cdot b + a \cdot \overline{b}$	
ET inclusif XNOR		a b S 0 0 1 0 1 0 1 1 1 1 0 0	S = $a \odot b$ $= \overline{a} \cdot \overline{b} + a \cdot b$	

$$\boxed{\begin{aligned} \overline{x \cdot y} &= \overline{x} + \overline{y} \\ \overline{x + y} &= \overline{x} \cdot \overline{y} \end{aligned}}$$

$$\overline{\overline{a} + \overline{b}} = \overline{\overline{a}} \cdot \overline{\overline{b}} = a \cdot b$$

Propriétés des fonctions logiques de base :

$1+0=1$
$0+0=0$

$1+1=1$
$0+1=1$

$1+0=1$
$0+1=1$

Fonction OU	Fonction ET
$x + 0 = x$	$x.0 = 0$
$x + y = y + x$ (commutativité)	$x. y = y.x$ (commutativité)
$x + 1 = 1$	$x.1 = x$
$x + x = x$	$x.x = x$
$x + \bar{x} = 1$	$x.\bar{x} = 0$
$x + x.y = x.(1+y) = x.1 = x$	

$1.0=0$
$0.0=0$

$1.1=1$
$0.1=0$

$1.0=0$
$0.1=0$

Théorèmes de DEMORGAN :

$\overline{x+y} = \bar{x}.\bar{y}$
$\overline{x.y} = \bar{x}+\bar{y}$

$x+1=1$ $x+\bar{x}=1$ $x+x=x$ $a.b+a.b=a.b$ $x.1=x$

Simplification algébrique

1- Simplifier les expressions suivantes

a + a b = $a.(1+b) = a.1 = a$

$a+\bar{a}b$ = $a.1+\bar{a}.b = a.(b+\bar{b})+\bar{a}.b = a.b+a.\bar{b}+\bar{a}.b+a.b = a.(b+\bar{b})+b(\bar{a}+a) = a.1+b.1 = a+b$

(a + b) (a + c) = $a.a+a.c+b.a+b.c = a+a.c+b.a+bc = a.(1+c+b)+bc = a.1+bc = a+bc$

$\bar{x}\bar{z}\,\bar{w}+xy\bar{z}\bar{w}+\bar{z}\bar{w}$ = $\bar{z}.\bar{w}.(\bar{x}+x.y+1) = \bar{z}.\bar{w}.1 = \bar{z}.\bar{w}$

$\bar{z}w+xy+xzw+xyz+zw+x\bar{y}$ = $x(y+\bar{y})+w(\bar{z}+z)+xz(w+y) = x.1+w.1+xz.(w+y) = x+w+xz(w+y)$
$= x.[1+z(w+y)]+w$
$= x.1+w = x+w$

2- En appliquant les théorèmes de DE MORGAN , développer et simplifier s'il a lieu les expressions logiques suivantes :

A = $\overline{\overline{ab}+c}$ = $\overline{\overline{a.b}}.\bar{c} = a.b.\bar{c}$

3- Montrer que B=$\overline{b+(\overline{a.c})}$

B=$\overline{(a+b)(b+c)}$ = $\overline{(a+b)}+\overline{(b+c)} = (\bar{a}.\bar{b})+(\bar{b}.\bar{c}) = \bar{b}(\bar{a}+\bar{c}) = \bar{b}.(\overline{a.c}) = \overline{b+(a.c)}$

$\overline{x.y}=\bar{x}+\bar{y}$; $\overline{x+y}=\bar{x}.\bar{y}$

Simplification graphique par tableau de Karnaugh :

1- Soit la fonction logique suivante :

$$S = \overline{a}.\overline{b}.\overline{c}.\overline{d} + \overline{a}.\overline{b}.\overline{c}.d + \overline{a}.\overline{b}.c.\overline{d} + a.\overline{b}.\overline{d} + \overline{a}.b.\overline{c}$$

Compléter le tableau de Karnaugh associé à cette fonction.

cd \ ab	00	01	11	10
00	1	1	0	1
01	1	1	0	1
11	0	0	0	0
10	1	0	0	0

$a.\overline{b}.\overline{d}.c$

$a.\overline{b}.\overline{d}.\overline{c}$

$\overline{a}.b.\overline{c}.d$

$\overline{a}.b.\overline{c}.\overline{d}$

- **Recherche de l'équation simplifiée**

cd \ ab	00	01	11	10
00	0	0	1	1
01	0	1	1	1
11	0	1	1	1
10	0	0	0	0

H = $a \cdot \bar{c} + a \cdot d + b \cdot d$

groupement → les cases de "1"

Nbres de cases $\boxed{2^n}$ 1 – 2 – 4 – 8 ....

2- Déterminer l'équation pour chaque tableau :

cd \ ab	00	01	11	10
00	1	1	0	1
01	1	1	0	1
11	0	0	0	1
10	0	0	0	1

H1= $\bar{a} \cdot \bar{c} + a \cdot \bar{b}$

cd \ ab	00	01	11	10
00	1	0	0	1
01	1	0	0	1
11	1	0	1	1
10	1	0	1	1

H2= $\bar{b} + a \cdot c$

cd \ ab	00	01	11	10
00	1	1	0	1
01	0	0	0	0
11	0	0	0	0
10	1	0	0	1

H3= $\bar{a} \cdot \bar{c} \cdot \bar{d} + \bar{b} \cdot \bar{d}$

cd \ ab	00	01	11	10
00	0	1	1	0
01	1	0	0	1
11	1	0	0	1
10	0	1	1	0

H4=$\bar{b} . d + b . \bar{d}$

c \ ab	00	01	11	10
0	1	1	1	1
1	1	1	0	0

H5=$\bar{c} + \bar{a}$

c \ ab	00	01	11	10
0	0	1	1	1
1	1	1	1	1

H6=$a + b + c$